

**TELECOM**  
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# Transform coding and **JPEG**

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# Plan

## Introduction

- Definitions

## Transform coding

- Resource allocation

- TCD

- Discrete cosine transform

## JPEG

- Principles

- Examples

- Extensions



# Plan

Introduction

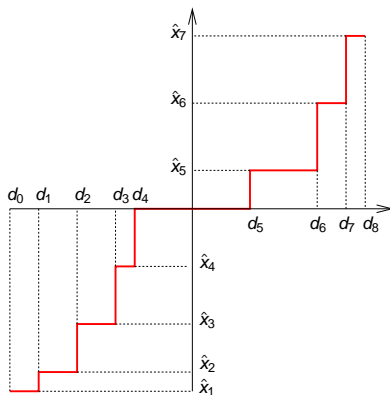
Definitions

Transform coding

JPEG

# Recall: Scalar quantization

$$Q : x \in \mathbb{R} \rightarrow y \in \mathcal{C} = \{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_L\} \subset \mathbb{R}$$



# Uniform Quantization



- ▶ Simple
- ▶ Minimize maximum error
- ▶ Optimal for uniform random variables (RV's)

RD curve for UQ of uniform RV:

$$D = \sigma_X^2 2^{-2R}$$

RD curve for UQ of non-uniform RV in *high resolution* :

$$D = K_X \sigma_X^2 2^{-2R}$$

# Uniform Quantization

## Example

Image Originale, 24 bpp



# Uniform Quantization

## Example

Débit 21 bpp PSNR 47.19 dB TC 1.143



# Uniform Quantization

## Example

Débit 18 bpp PSNR 42.38 dB TC 1.333





# Uniform Quantization

## Example

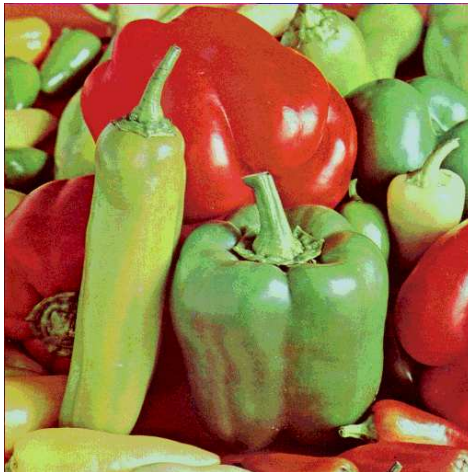
Débit 15 bpp PSNR 36.97 dB TC 1.600



# Uniform Quantization

## Example

Débit 12 bpp PSNR 31.40 dB TC 2.000



# Uniform Quantization

## Example

Débit 9 bpp PSNR 29.26 dB TC 2.667



# Uniform Quantization

## Example

Débit 6 bpp PSNR 27.83 dB TC 4.000



# Uniform Quantization

## Example

Débit 3 bpp PSNR 25.75 dB TC 8.000



# Quantization

## Conclusion

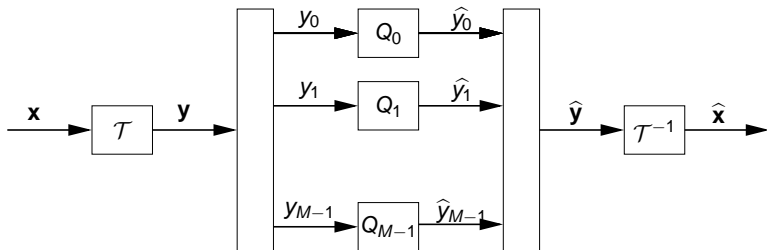
- ▶ Quantization: at the core of lossy coding
- ▶ Irreversible operation
- ▶ Central in the rate-distortion trade-off
- ▶ Approximations in HR:
  - ▶  $D \sim 2^{-2R}$
  - ▶  $D \sim \sigma^2$
- ▶ Quantization alone is not sufficient to give good RD performance

# Linear transform

- ▶ Linear transform is a base changement
  - ▶ Separation of data between important and irrelevant (energy concentration)
  - ▶ HSV relevant information
  - ▶ Resource allocation
- ▶ Correlation reduction

# Linear transform

Paradigm of transform coding



From  $\mathbf{x}$  to  $\mathbf{y} = \mathcal{T}\mathbf{x}$ : we want an “easier” vector for coding, i.e. with a few large coefficients and many small ones



# Orthogonal Transforms

It means that  $\mathcal{T}^{-1} = \mathcal{T}^T$

advantages :

1. immediate inversion
2. norm conservation  $\|\mathbf{Y}\| = \|\mathbf{X}\|$

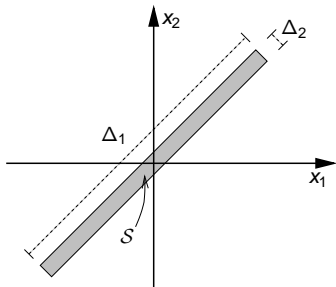
$\Rightarrow$  Distorsion sur  $\mathbf{Y}$  = distorsion sur  $\mathbf{X}$  :

$$\begin{aligned} E \left[ \|\mathbf{Y} - \hat{\mathbf{Y}}\|^2 \right] &= E \left[ (\mathbf{Y} - \hat{\mathbf{Y}})^T (\mathbf{Y} - \hat{\mathbf{Y}}) \right] = E \left[ (\mathbf{X} - \hat{\mathbf{X}})^T \mathcal{T}^T \mathcal{T} (\mathbf{X} - \hat{\mathbf{X}}) \right] \\ &= E \left[ \|\mathbf{X} - \hat{\mathbf{X}}\|^2 \right] \end{aligned}$$

# Transform coding

## Example

Couple of highly correlated RV's



$$f_{X_1, X_2}(x_1, x_2) = \begin{cases} \frac{1}{\Delta_1 \Delta_2} & \text{if } (x_1, x_2) \in S \\ 0 & \text{if } (x_1, x_2) \notin S \end{cases}$$

$$\Delta_1 \gg \Delta_2$$

$$X_1 \sim X_2 \sim \mathcal{U} \left[ -\frac{\Delta_1}{2\sqrt{2}}, \frac{\Delta_1}{2\sqrt{2}} \right]$$

# Transform coding

## Example

- ▶ Quantization of uniform variables  $\Rightarrow$  UQ
- ▶  $D_i(R_i) = \sigma_i^2 2^{-2R_i}$  for each RV
- ▶  $D = \sum_i D_i$
- ▶  $\sigma_1^2 = \sigma_2^2 = \sigma^2 = \left(\frac{\Delta_1}{\sqrt{2}}\right)^2 \frac{1}{12} = \frac{\Delta_1^2}{24}$

# Transform coding

## Example

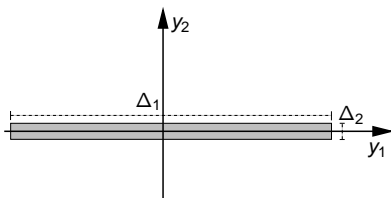
- ▶ Quantization of uniform variables  $\Rightarrow$  UQ
- ▶  $D_i(R_i) = \sigma_i^2 2^{-2R_i}$  for each RV
- ▶  $D = \sum_i D_i$
- ▶  $\sigma_1^2 = \sigma_2^2 = \sigma^2 = \left(\frac{\Delta_1}{\sqrt{2}}\right)^2 \frac{1}{12} = \frac{\Delta_1^2}{24}$

bits	$R$	$D_1$	$D_2$	$D$
0	0	$\sigma^2$	$\sigma^2$	$2\sigma^2$
1	0.5	$\sigma^2/4$	$\sigma^2$	$\frac{5}{4}\sigma^2$
1	0.5	$\sigma^2$	$\sigma^2/4$	$\frac{5}{4}\sigma^2$
2	1	$\sigma^2/4$	$\sigma^2/4$	$\frac{1}{2}\sigma^2$
2	1	$\sigma^2/16$	$\sigma^2$	$\frac{17}{16}\sigma^2$
3	1.5	$\sigma^2/16$	$\sigma^2/4$	$\frac{5}{16}\sigma^2$
4	2	$\sigma^2/16$	$\sigma^2/16$	$\frac{1}{8}\sigma^2$

# Transform coding

Transform: 45-degree rotation

After the transform the RV are independent



$$f_{Y_1, Y_2}(y_1, y_2) = \begin{cases} \frac{1}{\Delta_1 \Delta_2} & \text{if } (y_1, y_2) \in S \\ 0 & \text{if } (y_1, y_2) \notin S \end{cases}$$

$$Y_1 \sim \mathcal{U} \left[ -\frac{\Delta_1}{2}, \frac{\Delta_1}{2} \right]$$

$$Y_2 \sim \mathcal{U} \left[ -\frac{\Delta_2}{2}, \frac{\Delta_2}{2} \right]$$

# Transform coding

## Example

- ▶ Quantization of  $Y_1$  and  $Y_2$  :  $D(R)$  curve
- ▶  $\sigma_1^2 = \frac{\Delta_1^2}{12} = 2\sigma^2$
- ▶  $\sigma_2^2 = \frac{\Delta_2^2}{12} \ll \sigma^2$

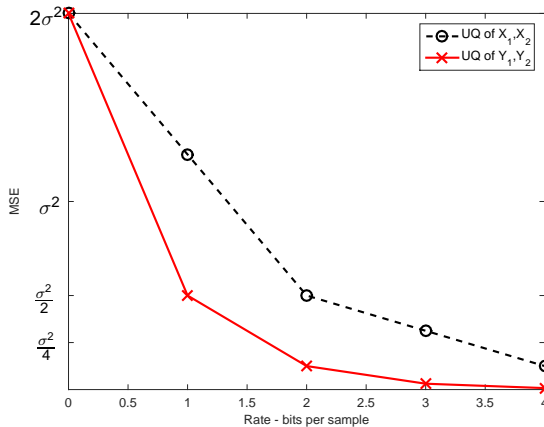
# Transform coding

## Example

- ▶ Quantization of  $Y_1$  and  $Y_2$  :  $D(R)$  curve
- ▶  $\sigma_1^2 = \frac{\Delta_1^2}{12} = 2\sigma^2$
- ▶  $\sigma_2^2 = \frac{\Delta_2^2}{12} \ll \sigma^2$

bits	$R$	$D_1$	$D_2$	$D$
0	0	$2\sigma^2$	$\ll \sigma^2$	$2\sigma^2$
1	0.5	$\sigma^2/2$	$\ll \sigma^2$	$\frac{1}{2}\sigma^2$
2	1	$\sigma^2/8$	$\ll \sigma^2$	$\frac{1}{8}\sigma^2$
3	1.5	$\sigma^2/32$	$\ll \sigma^2$	$\frac{1}{32}\sigma^2$

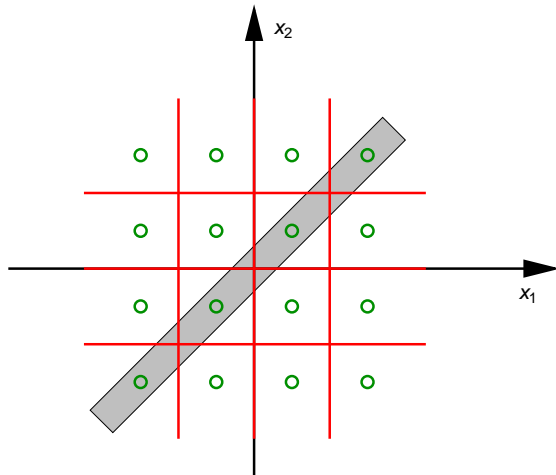
# Transform coding





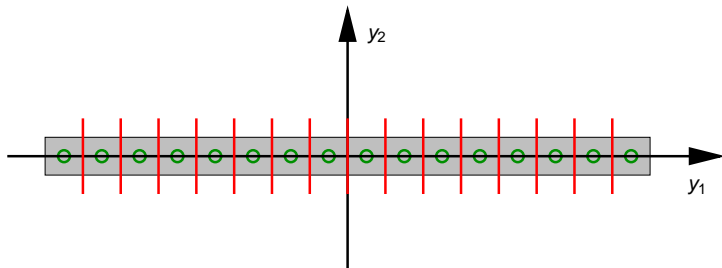
# Transform coding

Quantization regions: SQ



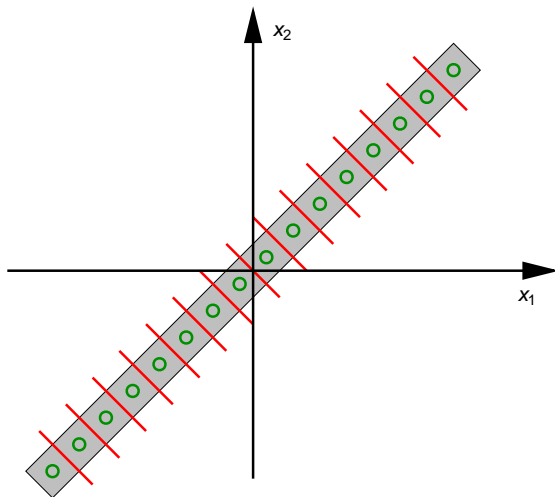
# Transform coding

Quantization regions:  $SQ+T$



# Transform coding

Quantization regions: VQ





# Plan

Introduction

Transform coding

Resource allocation

TCD

Discrete cosine transform

JPEG

## Resource allocation

- ▶ Components of  $\mathbf{y} = \mathcal{T}\mathbf{x}$  interpreted as realizations of  $M$  random stationary processes with variances  $\sigma_k^2$
- ▶ Case of orthogonal linear transform:  $\mathcal{T}$  is a square matrix and the distortion is the same in the two domain (original and transformed)

$$\begin{aligned}\mathcal{D} &= \text{E} \left[ \|\mathbf{X} - \hat{\mathbf{X}}\|^2 \right] = \text{E} \left[ \|\mathbf{Y} - \hat{\mathbf{Y}}\|^2 \right] \\ &= \text{E} \left[ \sum_{k=0}^{M-1} (Y_k - \hat{Y}_k)^2 \right] = \sum_{k=0}^{M-1} \text{E} \left[ (Y_k - \hat{Y}_k)^2 \right] \\ &= \sum_{k=0}^{M-1} \mathcal{D}_k\end{aligned}$$

- ▶ We know the relation between  $\mathcal{D}_k$  and the quantization rate  $b_k$  (HR hypothesis) :

$$\mathcal{D}_k = c_k \sigma_k^2 2^{-2b_k}$$

# Resource allocation

## Problem formulation

- ▶ Minimize  $\mathcal{D}$  under a rate constraint

$$\min \mathcal{D}(\mathbf{b}) = \sum_{k=0}^{M-1} c_k \sigma_k^2 2^{-2b_k} \quad \text{subject to} \quad \sum_{k=0}^{M-1} b_k \leq B$$

- ▶ Lagrange method. Minimize:

$$J(\mathbf{b}, \lambda) = \sum_{k=0}^{M-1} c_k \sigma_k^2 2^{-2b_k} + \lambda \left( \sum_{k=0}^{M-1} b_k - B \right)$$

- ▶ Solution (Huang-Schulteiss formula) :

$$b_k^* = \frac{B}{M} + \frac{1}{2} \log \left[ \frac{c_k \sigma_k^2}{c_{GM} \sigma_{GM}^2} \right]$$

# Resource allocations

## Interpretation

- ▶ A uniform resource allocation ( $\bar{b} = B/M$ ) is refined using variances
- ▶ Per-component distortion:

$$D_k^* = c_k \sigma_k^2 2^{-2b_k^*} = c_k \sigma_k^2 2^{-2\bar{b}} \frac{c_{GM} \sigma_{GM}^2}{c_k \sigma_k^2} = c_{GM} \sigma_{GM}^2 2^{-2\bar{b}}$$

$$D_T = \sum_{k=0}^{M-1} D_k^* = M c_{GM} \sigma_{GM}^2 2^{-2\bar{b}}$$

- ▶ Gaussian case:  $c_{GM} = c_k = c_N$  and thus

$$b_k^* = \bar{b} + \frac{1}{2} \log \left[ \frac{\sigma_k^2}{\sigma_{GM}^2} \right] \quad D_k^* = c_N \sigma_{GM}^2 2^{-2\bar{b}}$$

$$D_T = M c_N \sigma_{GM}^2 2^{-2\bar{b}}$$

## Coding gain

- ▶ Let  $X$  be a random vector of  $M$  components (sound, image...)
- ▶ Hypothesis :  $X$  components are i.d., e.g. Gaussian  $\mathcal{N}(0, \sigma_X^2)$
- ▶ Without transform, we have SQ and resource allocation. This case is referred to as PCM. The distortion is:

$$\mathcal{D}_{\text{PCM}} = Mc_{\mathcal{N}} \sigma_X^2 2^{-2\bar{b}}$$





## Coding gain

- ▶ The *Coding gain* of  $\mathcal{T}$  the ratio between the PCM distortion and the transform coding distortion:

$$G_{\mathcal{T}} = \frac{D_{\text{PCM}}}{D_{\mathcal{T}}} = \frac{\sigma_X^2}{\sigma_{\text{GM}}^2} = \frac{\sigma_{\text{AM}}^2}{\sigma_{\text{GM}}^2}$$

- ▶ The transform should make the vector components as different as possible
- ▶ The geometrical mean of a set of positive numbers is always less or equal than their arithmetical mean

# Resource allocations

## Practical Algorithms

### Huang-Schulteiss' formula

- ▶ might provide negative values
- ▶ might provide non-integer values

### Sub-optimal algorithms

- ▶ Modified HS (Huang-Schulteiss) Algorithm
- ▶ *Greedy* Algorithm

# Resource allocation

## Modified HS Algorithm

1. We compute  $b_k^*$  with Huang-Schulteiss ;
2. If some  $b_k^*$  are negative, we remove the variances of the concerned components and we repeat the computation.  
The removed variables are not coded (*i.e.*, coded with zero bits)
3. This step is repeated as long as there are negative allocation values.
4. The results are floored
5. The eventual residual rate is allocated to coefficients with the largest error

# Greedy algorithm

## 1. Initialization

- ▶  $b_k = 0 \quad \forall k \in \{0, 1, \dots, M-1\}$ .
- ▶  $D_k = \sigma_k^2 \forall k \in \{0, 1, \dots, M-1\}$ .

## 2. While $\sum_k b_k \leq B$

- ▶  $\ell = \arg \max_k D_k$
- ▶  $b_\ell \leftarrow b_\ell + 1$
- ▶  $D_\ell \leftarrow D_\ell/4$

## 2D linear transforms

- ▶ Asymptotically optimal when  $(N, M \rightarrow \infty)$

### DFT

$$t(k, \ell, n, m) = \frac{1}{\sqrt{NM}} \exp \left[ -i2\pi \left( \frac{(n-1)(k-1)}{N} + \frac{(m-1)(\ell-1)}{M} \right) \right]$$

### DCT

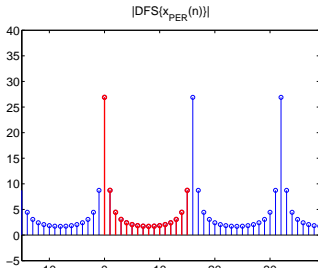
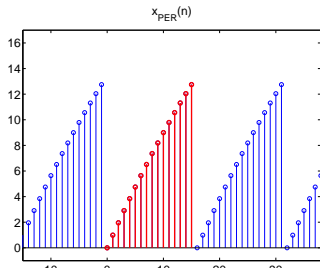
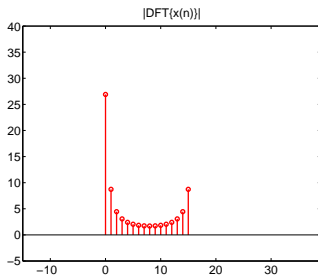
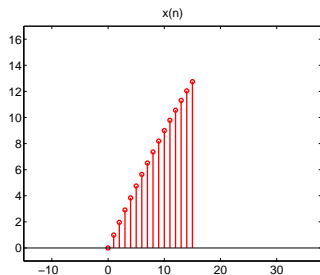
$$t(k, \ell, n, m) = \frac{c_k c_\ell}{\sqrt{NM}} \cos \left( \pi \frac{(2n-1)(k-1)}{2N} \right) \cos \left( \pi \frac{(2m-1)(\ell-1)}{2M} \right)$$
$$c_k = \begin{cases} 1 & \text{si } k = 1 \\ \sqrt{2} & \text{sinon} \end{cases}$$

## 2D linear transforms

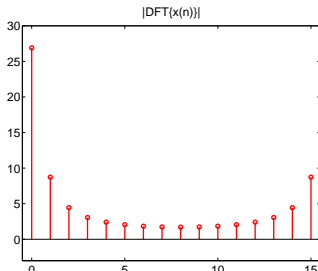
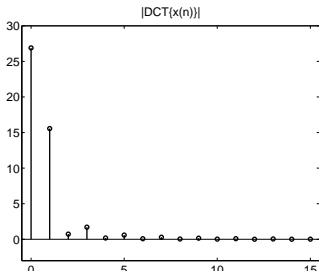
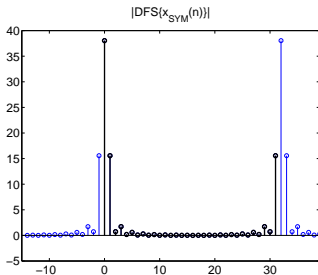
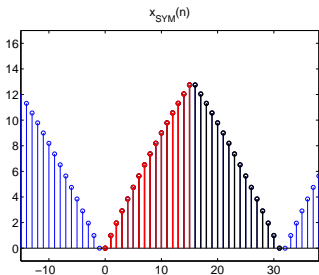
### DCT and DFT

- ▶ Common characteristics
  - ▶ separable :  $t(k, \ell, n, m) = t_1(k, n) t_2(\ell, m)$
  - ▶ fast computations
  - ▶ frequency interpretation
    - $k$  : vertical frequency
    - $\ell$  : horizontal frequency
- ▶ DCT characteristics
  - ▶ Real
  - ▶ Better energy concentration than DFT for images

# DFT vs DCT



# DFT vs DCT

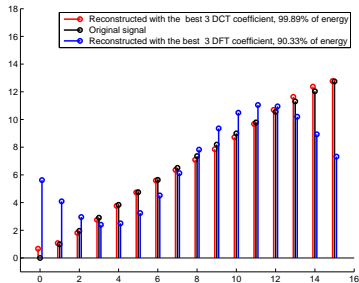




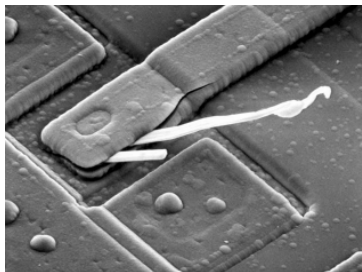
# DFT vs DCT

DCT	
Coeff	% Energy
1	74.61
2	24.98
3	0.05
4	0.30
5	0.00
6	0.04
7	0.00
8	0.01
9	0.00
10	0.00
11	0.00
12	0.00
13	0.00
14	0.00
15	0.00
16	0.00

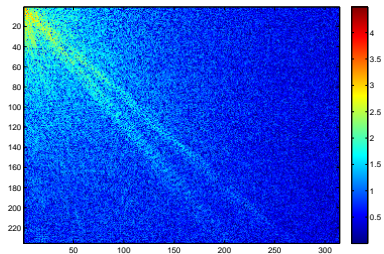
DFT	
Coeff	% Energy
1	74.61
2	7.86
3	2.04
4	0.97
5	0.60
6	0.43
7	0.35
8	0.31
9	0.30
10	0.31
11	0.35
12	0.43
13	0.60
14	0.97
15	2.04
16	7.86



# DCT examples

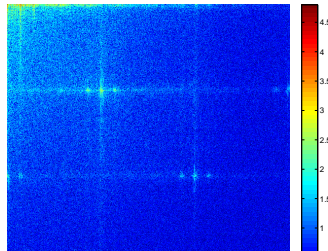
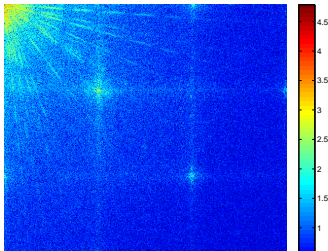
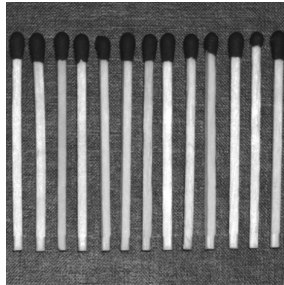
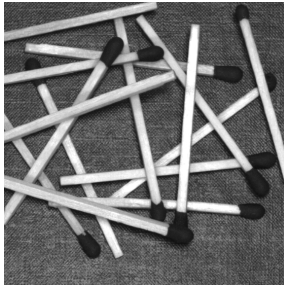


Image



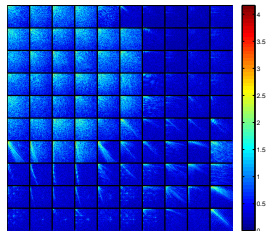
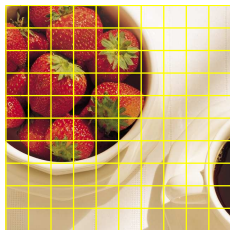
Logarithme des coefficients DCT

# DCT examples



## DCT par blocs

- ▶ Large-size non-stationary image  
 ⇒ divided into  $I \times J$  rectangular blocks  $(\mathcal{B}_{i,j})_{0 \leq i < I, 0 \leq j < J}$ , of size  $K \times L$  ( $K = L = 8$ )



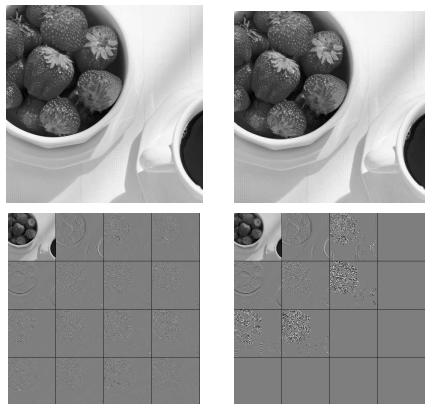
$$N = IK$$

$$M = JL$$

## Resource allocation for DCT coefficients

- ▶ Huang-Schulteiss formula, or
- ▶ Greedy Algorithm, or
- ▶ Fixed allocation (independent from data)
  - ▶ Very simple solution
  - ▶ Very common in practice (e.g., quantization matrix in JPEG)

## Resource allocation for DCT coefficients



sb	variance	lev	rate	dist
1	141.35	32	3.90	0.86
2	32.92	15	2.74	0.98
3	6.80	5	1.50	1.24
4	1.77	1	0.00	1.77
5	38.07	15	2.74	1.14
6	10.27	7	1.81	1.12
7	3.89	3	1.15	1.41
8	1.19	1	0.00	1.19
9	7.47	5	1.50	1.37
10	3.76	3	1.15	1.37
11	2.07	1	0.00	2.07
12	0.71	1	0.00	0.71
13	1.95	1	0.00	1.95
14	1.27	1	0.00	1.27
15	0.74	1	0.00	0.74
16	0.32	1	0.00	0.32



# Plan

Introduction

Transform coding

**JPEG**

Principles

Examples

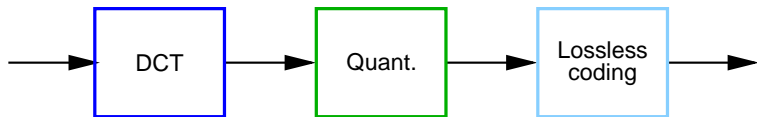
Extensions

# JPEG Standard

- ▶ Image compression standard based on DCT
- ▶ Specified in 1991, adopted in 1992
- ▶ It describes *only the decoder*
- ▶ It can work on multi-component (color) images, but we will describe its operation on one component, the other being identical



## JPEG Standard: Scheme



- ▶ The image is decomposed in  $8 \times 8$  pixels blocks
- ▶ The value 128 is subtracted from the image
- ▶ Blocks are (almost) independently encoded

 JPEG Standard: Blocks

Example of 8x8 block

173	171	171	143	109	100	91	96
171	169	150	137	112	101	94	96
184	158	139	120	110	107	94	100
170	156	134	119	117	104	98	99
157	147	125	127	103	109	90	98
149	146	132	120	113	107	101	93
147	141	119	119	111	101	100	92
160	122	117	116	115	116	102	95

## JPEG Standard: Transform

- ▶ DCT is performed on  $8 \times 8$  blocks
- ▶ Small blocks ensures stationarity
- ▶ Large blocks better exploit correlation
- ▶ Size chosen after experiences
- ▶ Coefficients DCT : impact SVH

 JPEG Standard: Transform

Coefficients DCT of the block

985.3	186.2	34.1	11.6	7.3	1.6	4.9	-8.2
40.3	47.8	5.7	-26.0	-5.3	-3.5	4.0	-1.0
6.3	4.0	-9.3	-6.7	-1.2	8.1	3.4	4.1
-0.0	4.9	-13.3	-20.8	-10.4	-1.0	-4.5	-5.1
2.1	-1.3	-1.6	0.6	3.6	3.3	8.1	-1.7
1.3	3.7	2.4	-2.7	-2.2	-3.0	-4.1	7.8
5.1	0.4	3.1	4.8	-1.4	2.5	9.8	5.3
-5.6	1.6	4.4	0.1	3.3	2.3	4.3	-8.4

# JPEG Standard: Transform

Standard deviation of DCT coefficients for natural images

396.64	100.99	49.26	31.15	19.74	14.57	8.76	7.33
100.23	55.78	37.40	24.77	16.44	11.70	8.44	6.25
49.42	36.39	28.01	20.40	14.64	10.46	7.64	5.88
30.82	24.05	19.73	15.47	11.99	8.88	6.83	5.45
21.09	16.79	14.79	11.54	9.19	7.30	5.90	4.68
15.32	11.91	10.31	8.71	7.15	5.78	4.61	3.91
11.22	8.58	7.66	6.78	5.69	4.64	3.82	3.24
8.21	6.65	5.93	5.52	4.45	3.75	3.15	2.80

## JPEG Standard: Quantization

- ▶ Uniform dead-zone quantization
- ▶  $\tilde{C}_{i,j} = \left\lfloor \frac{C_{i,j}}{q_{i,j}} \right\rfloor$
- ▶ The Quantization  $q$  table defines the RD trade-off, but it is not defined by the standard
- ▶ Thus  $q$  must be encoded
- ▶ Quality factor  $Q$

# JPEG Standard: Quantization

Quantization table example

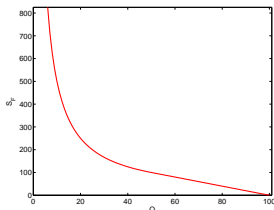
$$q^* =$$

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	81	61
18	22	37	56	68	109	103	77
24	35	55	64	81	104	111	90
49	63	78	87	101	121	120	100
72	92	95	98	112	100	103	99

## JPEG Standard: Quality factor

- ▶ Non normative tool
- ▶ Q between 1 and 100
- ▶ Defines a scaling factor  $S_F$  for the quantization matrix

$$S_F = \begin{cases} \frac{5000}{Q} & 1 \leq Q \leq 50 \\ 200 - 2Q & 50 < Q \leq 99 \\ 1 & Q = 100 \end{cases}$$



$$q \leftarrow \frac{S_F q^* - 50}{100}$$

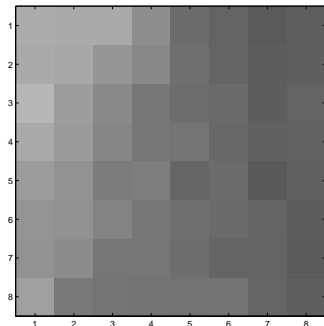


# JPEG Standard: Quantization

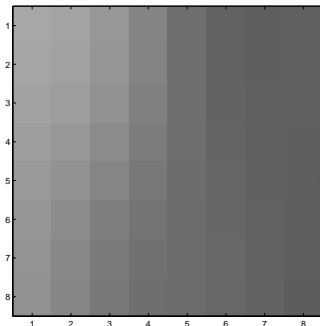
Quantized coefficients

61	16	3	0	0	0	0	0
3	3	0	-1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

# JPEG Standard: Example

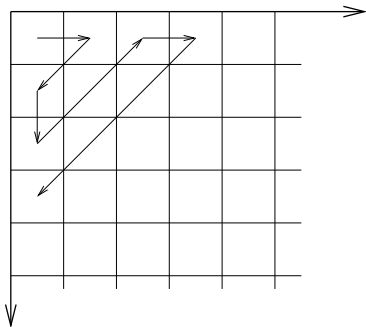


Original



$DCT \rightarrow Q \rightarrow Q^{-1} \rightarrow DCT^{-1}$

# JPEG Standard: Entropy coding



- ▶ Zig-zag scan

# JPEG Standard: Entropy coding

- ▶ DC Coefficient : prediction + Huffman
- ▶ AC Coefficients : “run-length” + Huffman

coeff $\neq$ 0	n. de 0	coeff $\neq$ 0	n. de 0	...	EOB	...
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# JPEG Standard: Entropy coding

Encoded quantized coefficients

61	16	3	0	0	0	0	0
3	3	0	-1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$61-dc_{k-1}$	0,16	0,3	1,3	0,3	7,-1	EOB
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## JPEG Standard: Entropy coding

### DC Coefficients :

- ▶ represented by the the category/amplitude couple
- ▶ There are 12 categories, named  $0, \dots, 11$ , coded on 4 bit
- ▶ Category  $k$  contains  $2^k$  values:  $\{\pm 2^{k-1}, \dots, \pm 2^k - 1\}$  ; each value is coded on  $k$  bits

### AC Coefficients:

- ▶ represented by run-length, category, amplitude.
- ▶ Categories and run-lengths are coded on 4 bit each: 8 bits in total for (R,C)
  - ▶ Special symbol 1: (15,0) means “at least 15 zeros before next non-zero coefficient”
  - ▶ Special symbol 2: (0,0) means “end of block”
- ▶ As in the DC case, the category  $k$  contains  $2^k$  values, each coded on  $k$  bits

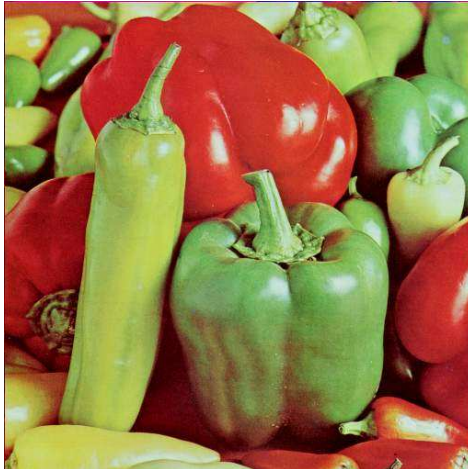
# JPEG Standard: Coding example

Image Originale, 24 bpp



# JPEG Standard: Coding example

Débit 1.0198674.2 bpp PSNR 33.92 dB TC 23.532





# JPEG Standard: Coding example

Débit 0.7481384.2 bpp PSNR 33.45 dB TC 32.080



# JPEG Standard: Coding example

Débit 0.5017404.2 bpp PSNR 32.70 dB TC 47.834



# JPEG Standard: Coding example

Débit 0.3081364.2 bpp PSNR 31.31 dB TC 77.888



# JPEG Standard: Coding example

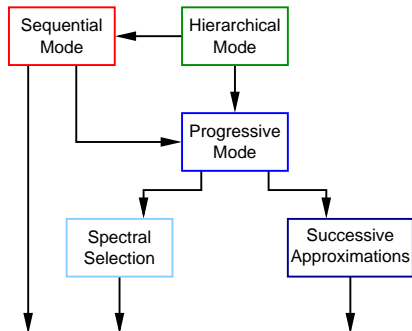
Débit 0.2069704.2 bpp PSNR 29.50 dB TC 115.959



## JPEG Standard: Parts

- ▶ JPEG baseline (sequential DCT mode)
- ▶ JPEG progressive
- ▶ JPEG hierarchical
- ▶ JPEG sequential lossless mode
- ▶ JPEG partie 3
  - ▶ Variable quantization
  - ▶ Tiling
- ▶ Standard JPEG-LS
- ▶ Motion JPEG

# JPEG Modes



# Progressive JPEG

## Spectral selection

Example with four quality layers

1. All the DC coefficients
2. The first three AC coefficients of all blocks (zig-zag scan)
3. AC coefficients 4 to 7 of all blocks
4. The other AC coefficients

JPEG syntax allow to define arbitrary layers, provided that they are made up of consecutive transform coefficients in the zig-zag scan order

The quality is the same as baseline JPEG, the rate might be slightly increased.

# Progressive JPEG

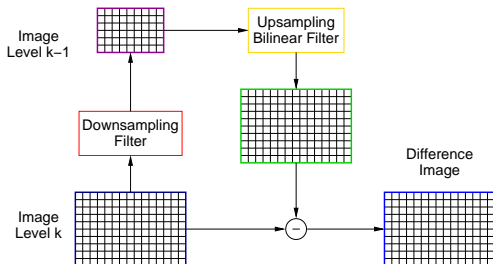
## Successive approximation

- ▶ First layer : all DC coefficients (as in Spectral Selection)
- ▶ Next layers: bit-plane coding of AC coefficients
  - ▶ An  $8 \times 8$  matrix of coefficients, represented on bits  $\Rightarrow 16$   
 $8 \times 8$  matrices of bits
  - ▶ Entropy coding of bitplanes
- ▶ Increased complexity
- ▶ RD performance slightly improved



# Hierarchical JPEG

## Pyramidal scheme



Each image is coded with baseline or progressive JPEG